

The Symmetries of Fermion Fluids at Low Dimensions

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We point out that the quasiparticle spectrum of the Landau Fermi liquid theory has an extra Z_2 symmetry, local in momentum space, which is not generic to the Hamiltonian with interactions. Thus the Fermi liquid is in this sense a (quantum) zero-temperature critical point.

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For many decades it has been accepted that the “natural” low-temperature state of a fermion fluid in the absence of a magnetic field is the Fermi Liquid described by Landau.⁽¹⁾ Landau’s fundamental assumption was that the states of an interacting electron gas (we shall hereafter call the fermions electrons) could be put into a one-to-one correspondence via adiabatic continuation with those of the free electron gas, an assumption which appears at first sight very plausible and is the basis of several derivations of the Fermi liquid theory.

Parenthetically, we note that it is accepted that in fact this is never actually the case at very low temperature, where a logarithmic divergence appears in one or another “Cooper channel” $k\sigma$, $-k - \sigma \rightarrow k'\sigma'$, $-k' - \sigma'$ and deforms the state into a BCS state for $D \geq 2$. But repulsive interactions are weakened by the Cooper channel divergence, while the most important attractive interactions, those due to phonons, are effective only at low energies, so that often the Fermi liquid has a wide range of validity.

We remark here that the Fermi liquid is, from a symmetry point of view, surprising, in that the Fermi liquid has an extra symmetry which is generically broken by the interactions. Only if the interaction term

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renormalizes to zero at low temperature is the Fermi liquid in principle the correct solution.

The symmetry of the kinetic energy terms for a free Fermi gas embodies the fact that there are separately conserved currents of both signs of the spin at every point in space—or, if we wish, separately conserved currents at each momentum. Landau's famous argument, following Weisskopf and others, is that scattering terms leave this conservation intact at the Fermi momentum due to exclusion principle prohibitions on final states. There are thus conserved currents of “up” and “down” spins where the quantization direction defining “up” is arbitrary at each momentum. There are thus two degenerate conserved complex fermions at each k_F , which we may describe in terms of four real (Majorana) fermions if we like.

Thus the fundamental symmetry is the group of real rotations among four objects, i.e., $O(4)$. $O(4)$ is isomorphous with the direct product of two $SU(2)$'s and the discrete group Z_2 . Z_2 occurs because the rotations of $O(4)$ may be divided into two classes, proper and improper rotations, depending on whether the determinant of the rotation is $+1$ or -1 . The simplest improper rotation just reverses the sign of one real fermion, which is equivalent to an electron-hole transformation for one spin. Thus $O(4)$ may be divided by Z_2 , and the resulting group of proper rotations is

$$O(4) \div Z_2 = SU(2) \times SU(2)$$

The two separate $SU(2)$ symmetries thus revealed are those of conserved charge current and conserved spin current. The idea of charge as an $SU(2)$ is a little unfamiliar but is behind the Anderson–Nambu Pauli operators τ_1, τ_2, τ_3 used in the BCS theory to describe the charge degrees of freedom of singlet pairs.⁽²⁾ The kinetic energy, for particles not at E_F , multiplies τ_3 and reduces the charge symmetry to $U(1)$.

The short-range repulsive interaction $Un_\uparrow n_\downarrow$ maximally violates the extra Z_2 symmetry of the Landau theory: it obviously changes sign under improper rotations, i.e., under hole-particle transformations for one spin only. This is an operation which interchanges charge and spin. But since charge and spin are conserved at every collision, the separate $SU(2)$ symmetries should be retained in the interacting fluid.

It has been assumed that U doesn't break the Z_2 symmetry of the Fermi liquid, since, as we said above, at the Fermi level the quasiparticle currents are perfectly conserved because of the exclusion principle, and apparently the effects of U have become irrelevant. But this is not the whole story: U modifies the density of states and the response coefficients, and in fact we know that the Landau interaction coefficients are not the same for charge and spin: the compressibility and spin susceptibility are

not related as they are for free particles because they contain Landau mean field corrections, and zero sound has a different velocity from spinwaves.

The assumption is, however, that at the level of single elementary excitations we still have the $O(4)$ or $U(2)$ symmetry with the extra Z_2 . In 1D it is known that this assumption fails: the Landau Fermi liquid gives way to the Luttinger liquid,⁽³⁾ which has only the separated charge and spin symmetries and has two separate velocities of elementary excitations. Analysis of the reason for this shows us that there is an interaction term which fails to renormalize to irrelevance in this case, namely the forward scattering phase shift for opposite-spin particles of zero relative momentum. This term effectively causes a chiral anomaly: the two currents of up and down spins are no longer separately conserved, rather there is a mixing of up-spin hole current with down-spin particles and vice versa.

In the conventional theory of the Fermi liquid in 3D this term in fact does renormalize to irrelevance because of the large phase space for recoil momenta: it is possible to renormalize the simple Born approximation by a simple partial wave analysis to make the scattering phase shift vanish proportionally to the relative momentum Q ⁽⁴⁾

$$\eta_0 = Qa$$

in 3 dimensions, where a is the effective scattering length used, especially, in the low density calculations of Huang and coworkers.⁽⁵⁾

2D is the critical dimensionality for this renormalization. This case was examined by Bloom⁽⁶⁾ and he showed that in the low-density limit it diverges as $1/Q |\ln Q|$, which gives a barely convergent theory. But as we have shown before, at any finite density the effective length theory does not renormalize the phase shift to zero and U can remain marginally relevant at the level of quasiparticle energies.

Thus in one and two dimensions the Fermi liquid can be thought of as a quantum critical point with an extra Z_2 symmetry, which is exact only at the singular point $U=0$ (or $n=0$). In general, no phase transition is required into the charge-spin separated state. In higher dimensionality, the Fermi liquid acquires the Z_2 symmetry continuously as $T \rightarrow 0$, which again is a kind of quantum critical point.

It is suggestive to think of the localized spin⁽⁷⁾ as a model for the above process. At $T \gg U$, the electron gas is effectively free and there is no local spin. As we go into the region $U > T \gg T_K$, this renormalizes into the Kondo model, which has a free electron gas interacting with a local spin (i.e., the interacting part of the gas has now separate charge and spin dynamics.)

Finally, as $T < T_K$ this renormalizes continuously away and we return to a quasi-particle resonance with no separate spin dynamics. The analogy is not perfect but it is suggestive. As Zlatic has shown,⁽⁸⁾ the low-temperature fixed point is analytically continuable from the high-temperature non-interacting state, yet the intermediate T states have no resemblance to either.

In conclusion, we have shown that the generic symmetry of the electron fluid involves separately conserved charge and spin currents. The quasiparticle concept implies a larger symmetry which in >2 dimensions manifests itself in the Fermi liquid, which is a $T=0$ quantum critical point. For 1D and perhaps 2 the $T=0$ state need not have this extra symmetry.

It is a pleasure, especially for the first author, to submit this paper to a volume in honor of Quin Luttinger, who was an old and valued friend and a superb physicist.

REFERENCES

1. L. D. Landau, *Sov. Phys. JETP* **3**:920 (1956).
2. P. W. Anderson, *Phys. Rev.* **112**:1900 (1958); Y. Nambu, *Phys. Rev.* **117**:648 (1960).
3. F. D. M. Haldane, *J. Phys. C* **14**:2585 (1981); E. Lieb and F. Y. Wu, *Phys. Rev. Lett.* **20**:1447 (1967).
4. K. Huang and C. N. Yang, *Phys. Rev.* **105**:767 (1958).
5. V. M. Galitskii, *Sov. Phys. JETP* **7**:104 (1958).
6. P. Bloom, *Phys. Rev. B* **12**:125 (1975).
7. F. D. M. Haldane, *Phys. Rev. Lett.* **40**:416 (1978); P. W. Anderson, *Phys. Rev.* **124**:1 (1959).
8. V. Zlatic and B. Horvatic, *Phys. Rev. B* **28**:6904 (1983).